Mastery Project 1

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6. Find the greatest common divisor of each of the following sets of integers.
a) 15, 35, 90 **b)** 300, 2160, 5040 **c)** 1240, 6660, 15540, 19980

8. Express the greatest common divisor of each set of numbers in *Exercise 6* as a linear combination of the numbers in that set.
$(a_1, a_2, \ldots, a_n) = c_1 a_1 + c_2 a_2 + \ldots + c_n a_n$

From 6:

a) $(15, 35, 90) = 5$ **b)** $(300, 2160, 5040) = 60$ **c)** $(1240, 6660, 15540, 19980) = 20$

Approach:

$$\begin{aligned}
&(a_1, a_2) & & = c_1 a_1 + c_2 a_2 \\
&(a_1, a_2, a_3) & = ((a_1, a_2), a_3) & = c_{12}(c_1 a_1 + c_2 a_2) + c_3 a_3 \\
&(a_1, a_2, a_3, a_4) & = (((a_1, a_2), a_3), a_4) & = c_{123}(c_{12}(c_1 a_1 + c_2 a_2) + c_3 a_3) + c_4 a_4 \\
& & \vdots &
\end{aligned}$$

Euclidean Algorithm:

Recall that $(a, b) = (a + cb, b)$

$$\begin{aligned}
&(a, b) & a = qb + r \\
=&(a - qb, b) & \\
=&(b, r) & \text{repeat} \\
=&(b_1, r_1) & \text{the} \\
=&(b_2, r_2) & \text{process} \\
&\vdots & \\
=&(b_n, r_n) & \\
=&(d, 0) & d = gcd(a, b)
\end{aligned}$$

Now, to find $d = c_1 a + c_2 b$.

$$\begin{aligned}
d = r_n =& b_{n-1} - q_{n-1} r_{n-1} \\
=& b_{n-1} - q_{n-1}(b_{n-2} - q_{n-2} r_{n-2}) \\
& \vdots \\
=& c_1 b_1 + c_2 r_1
\end{aligned}$$

Solutions:

$$\begin{array}{ll} (35,15) & q=2 \\ (15,5) & q=3 \\ (5,0) & \end{array}$$

$$\begin{array}{ll} (2160,300) & q=7 \\ (300,60) & q=5 \\ (60,0) & \end{array}$$

$$\begin{array}{ll} (6660,1240) & q=5 \\ (1240,460) & q=2 \\ (460,320) & q=1 \\ (320,140) & q=2 \\ (140,40) & q=3 \\ (40,20) & q=2 \\ (20,0) & \end{array}$$

$$5 = 1(35) - 2(15)$$

$$60 = 1(2160) - 7(300)$$

$$\begin{aligned} 20 &= 140 - 3(40) \\ &= 140 - 3(320 - 2(140)) \\ &= -3(320) + 7(140) \\ &= -3(320) + 7(460 - 320) \\ &= 7(460) - 10(320) \\ &= 7(460) - 10(1240 - 2(460)) \\ &= -10(1240) + 27(460) \\ &= -10(1240) + 27(6660 - 5(1240)) \\ &= 27(6660) - 145(1240) \end{aligned}$$

$$5 = -2(15) + 1(35) + 0(90)$$

$$60 = -7(300) + 1(2160) + 0(5040)$$

$$20 = -145(1240) + 27(6660) + 0(15540) + 0(19980)$$